MANAGEMENT TODAY
-for a better tomorrow
An International Journal of Management Studies
home page: www.mgmt2day.griet.ac.in
Vol.8, No.1, January-March 2018

Pricing of Stock Options using Black-Scholes, Black's and Binomial Option Pricing Models

Felcy R Coelho[1] and Y V Reddy[2]

[1]Assistant Professor, Department of Commerce, S.S.Dempo College of Commerce & Economics, Cujira, Goa and Research Scholar, Department of Commerce, Goa University, Goa, India. E-mail: felcycoelho@gmail.com

[2]Professor, Department of Commerce; and Registrar, Goa University, Taleigao Plateau, Goa 403206, India and is the corresponding author. E-mail: yvreddy@unigoa.ac.in

ARTICLE INFO

Article history:

Received 04.02.2018
Accepted 20.03.2018

Keywords:

option pricing, black & scholes, binomial, call option

Category: Research Paper

ABSTRACT

Derivative products are very important for developing the financial sector and generating economic growth. Among all the financial derivative products the most commonly used product is Options. Valuation of Options is very crucial in the modern financial industry. Most popular and largely accepted model for pricing options in the modern time is the model given by Black-Scholes. The present study attempts to examine whether the Black-Scholes, Black's and Binomial model is suitable for pricing call options on Indian stocks for five stocks selected from Financial Services Index of Nifty. Paired samples T-test results indicate that there exists a difference between the calculated prices using the three models and the market prices which is significant for four out of five cases while when the model prices calculated using three models and prices prevailing in the market were compared for HDFC Bank Ltd. stock no significant difference was found. The three models have effectively priced the call option premium of HDFC Bank Ltd. For all the five call options on stocks it was found that the models have underestimated option premium.

Introduction

An option is a contract, in which the buyer gets the right without the obligation, to purchase or sell a stated quantity of the underlying assets, at a stated price on or before the expiration date. Options may have physical commodities or financial instruments as its underlying asset. Options are of two types namely call and put option. The buyer of call option gets the right to purchase while in case of the put option he gets the right to sell. India witnessed a tremendous growth in financial derivatives market over the years. The Equity derivatives turnover on National Stock Exchange has increased from Rs. 2,365 cr in 2000-01 year to Rs. 9,43,70,301.61 cr in 2016-17. Among all the equity derivatives products the growth of index options has been phenomenal. Pricing Models of Options have occupied a significant place in Derivatives as appropriate pricing helps in computing a proper rate of premium which enables trading and helps to manage the risk. Correct pricing is crucial in order to decide whether to purchase or sell. There are many models for valuation of options which include Black-Scholes model, Black's, Binomial model, Hull and white model, Heston model etc. The Black-Scholes model is a milestone among the historical events of Derivatives. Black-Scholes model is the most accepted model to price options. This model is applied throughout the world by leading stock exchanges, investors and traders for pricing.

Attempts are made in order to alter the Black-Scholes model

ISSN: 2348-3989 (Online)
ISSN: 2230-9764 (Print)
Doi: http://dx.doi.org/10.11127/gmt.2018.03.21
pp. 103-108

as it results in the problem of deviation amongst the price prevailing in the market and calculated model price. Black used a forward price in the place of spot price in the original Black-Scholes model and developed another model for pricing. A comparatively easy and simple method of pricing is Binomial model but it is a powerful technique for valuing various kinds of options. Many researchers have tested the applicability of different models of option pricing and tried to find a model which is most suitable for Indian market. The present study tries to analyze the appropriateness of Black-Scholes model, Black's model and Binomial Model for pricing call options on Indian stocks for five stocks selected from Financial Services Index of Nifty. Financial services sector is a very crucial sector of Indian economy as it reflects the behavior and performance of the financial market. This sector is having the highest weightage among all the other sectors in the index of Nifty.

Literature Review

Black-Scholes (1973) derived a theoretical valuation formula for options and tested the valuation formula. The results revealed that actual prices of options vary from calculated model price. Black (1976) presented formulas for valuing forward contracts and options on commodities about the futures price and other variables. Cox, Ross and Rubinstein (1979) provided a simple distinct-time model to price options.

Tripathi & Gupta (2011) tested the predictive accuracy of pricing index options using Black-Scholes model to examine performance of the original BS model and skewness and kurtosis adjusted BS model of Corrado and Su (1996). They found out that the performance of the modified BS model is better than the original one. Panduranga (2013) has empirically tested the applicability of Black-Scholes model for select stock options from the banking sector. It was revealed from the results that the model has effectively priced the options as out of four banks three banks calculated model prices were close to the prices prevailing in the market.

In the papers of Mitra (2008 and 2012) a comparison was made between the calculated option prices using both Black's and Black-Scholes formulae with actual market prices. The results indicated that Black's model was superior in pricing performance than Black-Scholes model. Nagendran and Venkateswar (2014) tested the Black-Scholes model's validity for pricing Stock Options in India. The results showed that the Black-Scholes model is effective in pricing stock options and that if implied volatility is used in the model it results in improvement of its pricing efficiency. Gupta (2014) found that implied volatility predicts prices of options better in comparison to historical volatility. Fleming (1998) examined the capacity of the S&P 100 implied volatility to forecast stock market volatility of the future. It is observed from the results that the implied volatility is an upward biased forecast, and it also contains important information about future volatility.

Bolia and Juneja (2005) reviewed random tree methods, regression-based methods, and stochastic mesh methods and applied these models to find price of options. They have shown how important the sampling technique, may be combined with these methods to increase their effectiveness. Xiaoping and Jie (2014) studied the sensitiveness of put option price to the random binary tree parameters. The results exhibit that the effect of the occurrence probability of the random binomial tree environment on option prices is very significant.

Geske (1979) derived a new formula for the valuing compound option. It was observed in the study that some of the important biases of the Black-Scholes model were corrected by using the new model of pricing. Black, Derman and Toy (1990), described a model for valuing interest rate security and have shown the application of pricing options on Treasury Bonds. Moraleda and Pelsser (2000) tested various spot rate and forward rate models for cap and floor data and the findings showed that spot rate models performed better than the forward rate models. Miltersen and Schwartz (1998) developed a model for valuing commodity futures with stochastic interest rates and stochastic convenience yields. They have given numerical examples using Black-Scholes formulas.

Macbeth and Merville (1980) tested the Call option pricing model by Cox and Black-Scholes call option pricing model. They found that Cox call option pricing model is pricing more effectively than the Black–Scholes model. Scott (1987) scrutinized the pricing of European call options on stocks having variance rates that change randomly. They examined continuous time diffusion processes for the stock return and standard deviation parameter, and found that one should make use of the stock and two options in order to make riskless hedge.

Bennell and Sutcliffe (2004), made a comparison of Black–Scholes model and artificial neural network (ANN) for pricing European call options. ANN model allows for dividends. In the case of out-of-the-money options, the ANN performed better than Black–Scholes model. For in the money options, the results of the ANN is comparable to that of Black–Scholes. Amilon (2003) tried to find if option pricing is better using a neural network (MLP) than the Black–Scholes formula. The results revealed that the neural network models were superior in performance than the benchmarks with regards to pricing as well as hedging. The above studies show that various pricing models are applied which include Black-Scholes model, Black's, Binomial Model, Artificial neural network, etc. for different products like commodity futures, stock options, index options, interest rates, compound options etc. and attempts are made to examine the relevance of pricing models for various products.

This paper tries to study applicability of Black-Scholes model, Black's model and Binomial model for five stock call options from Nifty Financial Services Index to fill the gap in the existing literature. This is done by finding calculated option prices using the three models and using paired samples T-test to compare calculated model prices with the market prices.

Objectives

1. To ascertain the theoretical prices of stock options using Black-Scholes model, Black's model and Binomial Option Pricing Model
2. To find out whether there is significant difference between model prices and the actual market prices of stocks.

Hypotheses

Depending on the objectives of the study, the hypotheses are framed as follows:

H_o: There is no significant difference between the model prices and market prices of stocks.

H_1**:** There is a significant difference between the model prices and market prices of stocks.

Data and Methodology

The Nifty Financial Services index is having the highest weightage among all the sectoral indices of Nifty index. This index reflects how the financial market of India behaves and performs. The financial market comprises of banks, financial institutions, housing finance and other companies providing financial services. This Index can be used to benchmark fund portfolios, also to launch index funds, ETFs and structured products Study population constitutes of five call options on stocks which are ranked top five as per their weightage among the stocks of the Nifty Financial Services Index over a period of five years from April 2012 to March 2017. Deliberate Sampling method is used; the historical data are taken from the website of NSE.

Weighted average interest rate of the central government securities is used as proxy for risk free rate. Annualized volatility has been calculated on the basis of the daily closing prices of the previous financial year for each stock. Actual market prices of all the stocks for that year are used for comparing with the model prices.

Pricing is done in four weeks advance for two strike prices, one at In-the-Money (ITM) and the other one Out-of-the Money (OTM). Calculation of call prices is done for selected stocks using Black-Scholes Model, Black's and Binomial model. Paired samples T-test is utilized to make a comparison of the actual market prices and the theoretical model prices as per Black-Scholes, Blacks and Binomial option valuation model.

Black-Scholes Model

The Black-Scholes model of pricing stock options was established by Fischer Black and Myron Scholes. It is known all over the world and commonly used model for pricing options. The five variables which are used in the model include spot price, variance, strike price, risk free rate and time to expiry. The model uses the following equations for pricing Call options:

Call Option Premium

$C = SN(d_1) - Xe^{-rt}N(d_2)$

$$d1 = \frac{\left[\text{Ln}\left(\frac{S}{X}\right) + \left(r + \frac{\sigma 2}{2}\right) X\, t\right]}{\sigma \sqrt{t}}$$

or

$d_2 = d_1 - \sigma \sqrt{t}$

Where,

C = Call option price

S = Price of the underlying asset

X = Strike price of the option

r = Interest rate

t = Time to expiration

σ = Volatility of the underlying asset

N represents a standard normal distribution with mean = 0 and standard deviation = 1

Black's Model

He used forward price in place of spot price in the original model. He substituted the spot term (S) by the discounted value of future price $F.e^{-rt}$ in the original Black–Scholes Pricing formula. The Black's model uses the following equation for pricing call options:

$C = F.e^{-rt}N(d_1) – X.e^{-rt}(Nd_2)$

Binomial Model

Cox-Ross-Rubinstein introduced the binomial tree approach for pricing options in 1979. It is a simple, easy and flexible pricing model for pricing various kinds of derivatives. This model has a large popularity. In the study a three step binomial tree is considered for calculation of the price of option. The calculation is done as follows:

- Stock price is considered to be S at Time t. In Binomial model the price can either increase or decrease over the time. At time t_1 i.e. after one time interval the price can either increase or decrease. Since the possible prices "branch" out over a period of time this option valuation method is called as Binomial tree.
- Stock price is multiplied by the up ratio and the down ratio i.e. $S_u = S*u$ and $S_d = S*d$.

This calculation is constant throughout the tree.

$u = e\, \sigma \cdot \sqrt{\Delta t}$

$d = e^{-\sigma \cdot \sqrt{\Delta t}} = \frac{1}{u}$

$p = \frac{e^{r \cdot \Delta t} - d}{u - d}$

P is the probability of S moving up to S_u, 1-p is the probability of S coming down S_d.

Price at $t_1 = p.\ S_u + (1-p) . S_d$

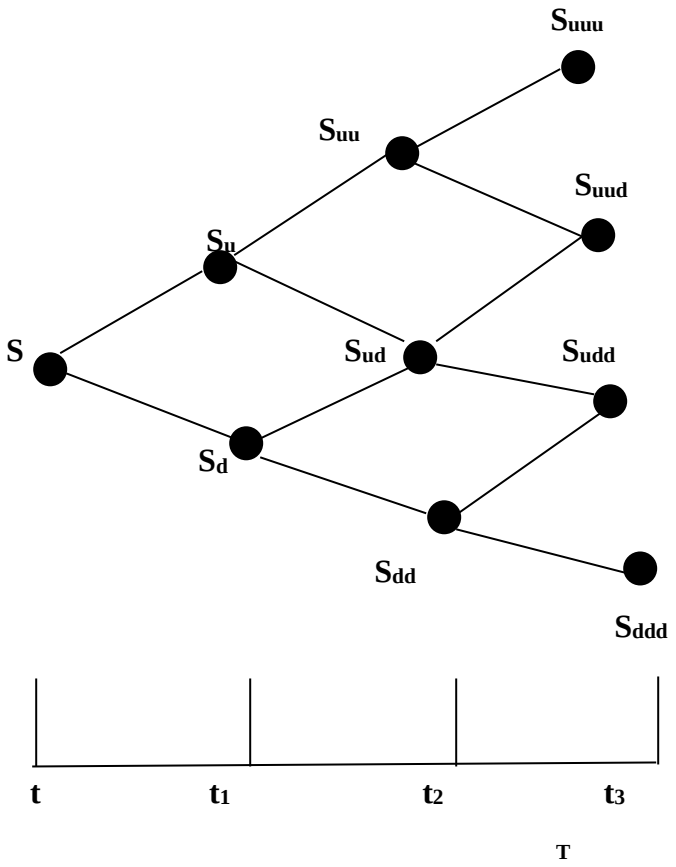
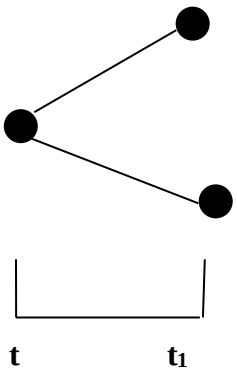


Fig 1: Three step Binomial Tree



For making the model risk neutral Binomial model, a riskless asset should grow by a factor of $e^{r.\ t}$ after delta $\triangle t$, with r as the risk free rate.

Price of S= $e^{-r.\triangle t}$ p. S_u + (1-p). S_d

Results and Discussion

Model prices are calculated by using Black-Scholes, Black's and Binomial option pricing model. The following table shows the results of paired samples t - test for five selected stocks from the Nifty Financial Services Index for five years period from April 2012 to March 2017. The paired sample t-test is done between the actual market prices and calculated premium of stock call option using Black Scholes Model, the actual market prices of stock call options and calculated premium using Black's option Pricing Model and calculated premium using Binomial option Pricing Model and the actual market prices of stock call options.

The Table 1 shows that there is significant difference between the calculated model price and actual price of HDFC, ICICI Bank Ltd., State Bank of India. And Kotak Mahindra Bank Ltd call options using all three pricing models. While the model price calculated using all the three models is very close to the actual price of the HDFC Bank Ltd Stock call options. This indicates that all the three models have effectively priced call option of HDFC Bank Ltd.

It is noticed that three models i.e. Black-Scholes, Black's and Binomial model are underestimating the premium for all the stocks call options under study.

Table 1: Comparison of Market Premiums and Model Estimate Premium as per Black-Scholes, Black's and Binomial Option pricing model

Stocks		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
HDFC Bank Ltd.	Market Premium –BS Model Premium	3.47283	20.04696	1.83003	-.15081	7.09647	1.898	119	.060
	Market Premium –B Model Premium	3.66167	19.65747	1.79447	.10843	7.21490	2.041	119	.044
	Market Premium –Bin Model Premium	2.40758	21.29900	1.94432	-1.44237	6.25754	1.238	119	.218
HDFC	Market Premium –BS Model Premium	1.30278E1	8.63304	.78809	11.46735	14.58832	16.531	119	.000
	Market Premium –B Model Premium	1.33324E1	8.37236	.76429	11.81905	14.84578	17.444	119	.000
	Market Premium –Bin Model Premium	1.28090E1	8.76473	.80011	11.22471	14.39329	16.009	119	.000
ICICI Bank Ltd.	Market Premium –BS Model Premium	9.55400	14.20769	1.29698	6.98585	12.12215	7.366	119	.000
	Market Premium –B Model Premium	9.55833	14.03003	1.28076	7.02230	12.09437	7.463	119	.000
	Market Premium –Bin Model Premium	8.94483	14.54712	1.32796	6.31533	11.57433	6.736	119	.000
Kotak Mahindra Bank Ltd.	Market Premium –BS Model Premium	7.30383	12.15543	1.10963	5.10665	9.50102	6.582	119	.000
	Market Premium –B Model Premium	8.68742	12.08167	1.10290	6.50356	10.87127	7.877	119	.000
	Market Premium –Bin Model Premium	7.92975	15.38188	1.40417	5.14936	10.71014	5.647	119	.000
State Bank of India	Market Premium –BS Model Premium	1.79308E1	25.93737	2.36775	13.24237	22.61913	7.573	119	.000
	Market Premium –B Model Premium	2.15800E1	26.34574	2.40503	16.81781	26.34219	8.973	119	.000
	Market Premium –Bin Model Premium	1.68260E1	26.27757	2.39880	12.07613	21.57587	7.014	119	.000

Conclusion

This study will help various sections from the financial market. This study of pricing of options will help the investors in understanding the functioning of option pricing. It will help

in understanding the Black- Scholes model, Black's model and Binomial model and their calculation to investors and traders as they make use of calculated option values to evaluate changing risk and to know the gain from their option positions, and will help them to take decisions relating to trading.

The results indicated that for four out of five cases there was significant difference between the calculated price using the three models and prices prevailing in the market which include the call options on HDFC, ICICI Bank Ltd., State Bank of India and Kotak Mahindra Bank Ltd Stocks. The data analysis shows that HDFC Bank Ltd. stock call options model prices calculated using all the three models are near to the actual market prices which shows that all the three models are effective in pricing HDFC Bank stock call options. For all the stock call options the three models used in the study have underestimated options premium.

Limitations

The present study covers only one sector i.e. Financial Services sector of Nifty and a sample is taken of only top five companies based on their weightage from this sector and only five years data are used for analysis. Further the study is confined to just three models of pricing options. The results may vary across sectors and with a large sample for an extended period of study.

References

1. Amilon, H. (2003). A Neural Network versus Black–Scholes: A Comparison of Pricing and Hedging Performances. *Journal of Forecasting*, J. Forecast. 22, 317–335.
2. Bennell, J. and Sutcliffe, C. (2004). Black–Scholes Versus Artificial Neural Networks In Pricing FTSE 100 options. *Intelligent Systems in Accounting Finance and*, 12, 243–260.
3. Black, F. (1976). The pricing of commodity contracts. *Journal of Financial Economics*, 3(1 & 2), 167-179.
4. Black, F., Derman, E. and Toy, W. (1990). A one-factor model of interest rates and its application to Treasury bond options. *Financial Analysts Journal*, 46, 33–39.
5. Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, 81(3), 637-654.
6. Bolia, N. and Juneja, S. (2005). Monte Carlo methods for pricing financial options. *Sadhana*, 30(2 & 3), 347–385.
7. Cox, J. C., Ross, S. A. and Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 7, 229-263.
8. Fleming, J. (1998). The quality of market volatility forecasts implied by S&P 100 index option prices. *Journal of Empirical Finance*, 5, 317–345.
9. Geske, R. (1979). The valuation of compound options. *Journal of Financial Economics*, 7, 63-81.
10. Gupta, S. (2014). Effectiveness of the Black and Scholes Model in pricing Nifty Call Options. *The International journal of Business and Management*, 2(7),149-158.
11. Macbeth, J. D. and Merville, L. J. (1980). Tests of the Black-Scholes and Cox Call Option Valuation Models. *Journal of finance*, 35(2), 285-301.
12. Miltersen, K. R. and Schwartz, E. S. (1998). Pricing of options on commodity futures with stochastic term structures of convenience yields and interest rates. *Journal of Financial and Quantitative Analysis*, 33(1), 33–59.
13. Mitra, S. K. (2008). Valuation of Nifty Options Using Black's Option Pricing Formula. *The Icfai Journal of Derivatives Market*, 5(1), 50-61.
14. Mitra, S. K. (2012). Pricing of Index Options Using Black's Model. *Global Journal of Management and Business Research*, 12(3), 89-96.
15. Oraleda, J. M. and Pelsser, A. (2000). Forward versus spot interest rate models of the term structure: An empirical comparison. *The Journal of Derivatives*, 7, 9–21.
16. Nagendran, R. and Venkateshwar, S. (2014). Validating Black–Scholes model in pricing Indian stock call options. *Journal of Applied Finance and Banking*, 4(3), 89-101.
17. Panduranga, V. (2013). An Empirical analysis of Black–Scholes option pricing model for select Banking stocks. *Vidyaniketan Journal of Management and Research*, 1(2), 23-30.
18. Scott, L. O. (1987). Option Pricing when the Variance Changes Randomly: Theory, Estimation, and an Application. *Journal of Financial and Quantitative Analysis*, 22(4), 419-438.
19. Tripathi, V. and Gupta, S. (2011). Effectiveness of the Skewness and Kurtosis adjusted Black- Scholes model in pricing Nifty call options. *The Indian Journal of Commerce*, 64(4), 17-30.
20. Xiaoping , H. and Jie, C. (2014). Randomized Binomial Tree and Pricing of American-Style Options. *Mathematical Problems in Engineering*, 2014, 1-6.